

Cross Product Formal Properties of Vector Algebra

- Commutative Property $\mathbf{u} \times \mathbf{v} = -\mathbf{v} \times \mathbf{u}$
- Associative Property $s(\mathbf{u} \times \mathbf{v}) = (s\mathbf{u}) \times \mathbf{v} = \mathbf{u} \times (s\mathbf{v})$
- Distributive Property $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) + (\mathbf{u} \times \mathbf{w})$
- Zero Property $\mathbf{u} \times \mathbf{0} = \mathbf{0} \times \mathbf{u} = \mathbf{0}$
- Self-Orthogonality $\mathbf{u} \times \mathbf{u} = \mathbf{0}$

★ **Example 3.8: Cross Product Calculation.** Returning to the simple numerical example from before, we now calculate the cross product instead of the inner product:

$$\begin{aligned} \mathbf{u} \times \mathbf{v} &= [3, 3, 3] \times [1, 2, 3] \\ &= [(3)(3) - (3)(2), (3)(1) - (3)(3), (3)(2) - (3)(1)] = [3, -6, 3]. \end{aligned}$$

We can then check the orthogonality as well:

$$[3, 3, 3] \cdot [3, -6, 3] = 0 \qquad [1, 2, 3] \cdot [3, -6, 3] = 0.$$

Sometimes the distinction between row vectors and column vectors is important. While it is often glossed over, vector multiplication should be done in a conformable manner with regard to multiplication (as opposed to addition discussed above) where a row vector multiplies a column vector such that their adjacent “sizes” match: a $(1 \times k)$ vector multiplying a $(k \times 1)$ vector for k