

and $\mathbf{v} = [1, 1, 1]$. As with the more basic addition and subtraction or scalar operations, there are formal properties for inner products:

Inner Product Formal Properties of Vector Algebra

$$\rightarrow \text{Commutative Property} \quad \mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$$

$$\rightarrow \text{Associative Property} \quad s(\mathbf{u} \cdot \mathbf{v}) = (s\mathbf{u}) \cdot \mathbf{v} = \mathbf{u} \cdot (s\mathbf{v})$$

$$\rightarrow \text{Distributive Property} \quad (\mathbf{u} + \mathbf{v}) \cdot \mathbf{w} = \mathbf{u} \cdot \mathbf{w} + \mathbf{v} \cdot \mathbf{w}$$

$$\rightarrow \text{Zero Property} \quad \mathbf{u} \cdot \mathbf{0} = 0$$

$$\rightarrow \text{Unit Rule} \quad \mathbf{1}\mathbf{u} = \mathbf{u}$$

$$\rightarrow \text{Unit Rule} \quad \mathbf{1}\mathbf{u} = \sum_{i=1}^k \mathbf{u}_i, \text{ for } \mathbf{u} \text{ of length } k$$

★ **Example 3.7: Vector Inner Product Calculations.** This example demonstrates the first three properties above. Define $s = 5$, $\mathbf{u} = [2, 3, 1]$, $\mathbf{v} = [4, 4, 4]$, and $\mathbf{w} = [-1, 3, -4]$. Then:

$s(\mathbf{u} + \mathbf{v}) \cdot \mathbf{w}$	$s\mathbf{v} \cdot \mathbf{w} + s\mathbf{u} \cdot \mathbf{w}$
$5([2, 3, 1] + [4, 4, 4]) \cdot [-1, 3, -4]$	$5[4, 4, 4] \cdot [-1, 3, -4]$
$5([6, 7, 5]) \cdot [-1, 3, -4]$	$+5[2, 3, 1] \cdot [-1, 3, -4]$
$5([6, 7, 5]) \cdot [-1, 3, -4]$	$[20, 20, 20] \cdot [-1, 3, -4]$
$[30, 35, 25] \cdot [-1, 3, -4]$	$+ [10, 15, 5] \cdot [-1, 3, -4]$
-25	$-40 + 15$
-25	-25