go 1 unit along the x-axis toward the point at 20 and construct line segments connecting the points along the curve at these x levels. The slope of the line segment (easily calculated from Section 1.5.1) is therefore an approximation to the instantaneous rate at $x = 20$, "rise-over-run," given by the segment

$$m = \frac{f(x_2) - f(x_1)}{x_2 - x_1}.$$

So the first line segment of interest has values $x_1 = 16$ and $x_2 = 24$. If we call the width of the interval $h = x_2 - x_1$, then the point of interest, x, is at the center of this interval and we say

$$m = \frac{f\left(x + \frac{h}{2}\right) - f\left(x - \frac{h}{2}\right)}{h}$$

$$= \frac{f(x + h) - f(x)}{h}$$

because $f(h/2)$ can move between functions in the numerator. This segment is shown as the lowest (longest) line segment in the second panel of Figure 5.3 and has slope 2.6625.

In fact, this estimate is not quite right, but it is an average of a slightly faster rate of change (below) and a slightly slower rate of change (above). Because this is an estimate, it is reasonable to ask how we can improve it. The obvious idea is to decrease the width of the interval around the point of interest. First go to 17–23 and then 18–22, and construct new line segments and therefore new estimates as shown in the second panel of Figure 5.3. At each reduction in interval width we are improving the estimate of the instantaneous rate of change at $x = 20$. Notice the nonlinear scale on the y-axis produced by the curvature of the function.

When should we stop? The answer to this question is found back in the previous discussion of limits. Define h again as the length of the intervals created as just described and call the expression for the slope of the line segment $m(x)$, to distinguish the slope form from the function itself. The point where $\lim_{h \to 0}$ occurs is the point where we get ***exactly*** the instantaneous rate of change at $x = 20$ since the width of the interval is now zero, yet it is still "centered"