Therefore we can get a matrix

$$\mathbf{A}^* = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \gamma_{24} & 1 & \gamma_{26} & 0 & -1 \\ 0 & -1 & \gamma_{56} & 0 & 0 \\ 0 & \gamma_{65} & -1 & 0 & 0 \\ \beta_{34} & 0 & \beta_{36} & 0 & \beta_{32} \\ \beta_{44} & 0 & \beta_{46} & 0 & \beta_{42} \end{bmatrix}'$$

$$= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & \gamma_{65} & 0 & 0 \\ \gamma_{26} & \gamma_{56} & -1 & \beta_{36} & \beta_{46} \\ \gamma_{24} & 0 & 0 & \beta_{34} & \beta_{44} \\ -1 & 0 & 0 & \beta_{32} & \beta_{42} \end{bmatrix}$$

that is immediately identifiable as having a zero determinant by the general determinant form given on page 142 because each ith minor (the matrix that remains when the ith row and column are removed) is multiplied by the ith value on the first row.

Some rank properties are more specialized. An idempotent matrix has the property that

$$\text{rank}(\mathbf{X}) = \text{tr}(\mathbf{X}),$$

and more generally, for any square matrix with the property that $A^2 = sA$, for some scalar s

$$s\text{rank}(\mathbf{X}) = \text{tr}(\mathbf{X}).$$

To emphasize that matrix rank is a fundamental principle, we now give some standard properties related to other matrix characteristics.