distinction, we say that $\mathbf{X}$ **pre-multiplies** $\mathbf{Y}$ or, equivalently, that $\mathbf{Y}$ **post-multiplies** $\mathbf{X}$.

So how is matrix multiplication done? In an attempt to be somewhat intuitive, we can think about the operation in ***vector terms***. For $\mathbf{X}_{k\times n}$ and $\mathbf{Y}_{n\times p}$, we take each of the n row vectors in $\mathbf{X}$ and perform a vector inner product with the n column vectors in $\mathbf{Y}$. This operation starts with performing the inner product of the first row in $\mathbf{X}$ with the first column in $\mathbf{Y}$ and the result will be the first element of the product matrix. Consider a simple case of two arbitrary 2×2 matrices:

$$\begin{aligned}
\mathbf{XY} &= \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \\
&= \begin{bmatrix} (x_{11}\ x_{12}) \cdot (y_{11}\ y_{21}) & (x_{11}\ x_{12}) \cdot (y_{12}\ y_{22}) \\ (x_{21}\ x_{22}) \cdot (y_{11}\ y_{21}) & (x_{21}\ x_{22}) \cdot (y_{12}\ y_{22}) \end{bmatrix} \\
&= \begin{bmatrix} x_{11}y_{11} + x_{12}y_{21} & x_{11}y_{12} + x_{12}y_{22} \\ x_{21}y_{11} + x_{22}y_{21} & x_{21}y_{12} + x_{22}y_{22} \end{bmatrix}.
\end{aligned}$$

Perhaps we can make this more intuitive visually. Suppose that we notate the four values of the final matrix as $\mathbf{XY}[1,1]$, $\mathbf{XY}[1,2]$, $\mathbf{XY}[2,1]$, $\mathbf{XY}[2,2]$ corresponding to their position in the 2×2 product. Then we can visualize how the rows of the first matrix operate against the columns of the second matrix to produce each value:

$$\begin{bmatrix} x_{11} & x_{12} \end{bmatrix} \begin{bmatrix} y_{11} \\ y_{21} \end{bmatrix} = \mathbf{XY}[1,1], \qquad \begin{bmatrix} x_{11} & x_{12} \end{bmatrix} \begin{bmatrix} y_{12} \\ y_{22} \end{bmatrix} = \mathbf{XY}[1,2],$$